Name:

# Fall 2018 Math 245 Final Exam

Please read the following directions:
Please write legibly, with plenty of white space. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use a single page of notes, but no calculators or other aids. This exam will last 120 minutes; pace yourself accordingly. Please try to keep a quiet test environment for everyone. Good luck!

| Problem | Min Score | Your Score | Max Score |
|---|---|---|---|
| 1. | 5 | | 10 |
| 2. | 5 | | 10 |
| 3. | 5 | | 10 |
| 4. | 5 | | 10 |
| 5. | 5 | | 10 |
| 6. | 5 | | 10 |
| 7. | 5 | | 10 |
| 8. | 5 | | 10 |
| 9. | 5 | | 10 |
| 10. | 5 | | 10 |
| 11. | 5 | | 10 |
| 12. | 5 | | 10 |
| 13. | 5 | | 10 |
| 14. | 5 | | 10 |
| 15. | 5 | | 10 |
| 16. | 5 | | 10 |
| 17. | 5 | | 10 |
| 18. | 5 | | 10 |
| 19. | 5 | | 10 |
| 20. | 5 | | 10 |
| Total: | 100 | | 200 |

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. Double Negation semantic theorem

b. disjunctive syllogism

c. contrapositive

d. proof by contradiction

Problem 2. Carefully define the following terms:

a. proof by induction

b. big O

c. union

d. disjoint

REMINDER: Use complete sentences.

Problem 3. Carefully define the following terms:

a. partition

b. transitive

c. Modular Division theorem

d. Chinese Remainder theorem

Problem 4. Carefully define the following terms:

a. product order

b. upper bound

c. height

d. surjection

Problem 5. Let $x \in \mathbb{R}$. Suppose that $x$ is not odd. Prove that $\frac{x}{3}$ is not odd.

Problem 6. Find all integers $x \in [0, 18)$ satisfying $15x \equiv 6 \pmod{18}$.

Problem 7. Relation $R = \{(a, b) : a^2 | b^3\}$ on $\{1, 2, 3, 4, 5, 6, 7\}$ is a partial order. Draw its Hasse diagram.

Problem 8. Prove that relation $R = \{(a, b) : a^2 | b^3\}$ on $\mathbb{N}$ is *not* an partial order.

Problem 9. Let $S, T$ be sets. Carefully state the converse of: If $S \subseteq T$, then $S \cap T = S$. Then, prove or disprove your statement.

Problem 10. Consider relation $S = \{(a, b) : a \leq b^2 + b\}$ on $\mathbb{R}$. Prove or disprove that $S$ is reflexive.

Problem 11. Prove that $\sqrt{3}$ is irrational.

Problem 12. Let $S$ be a set. Prove that $S_{diagonal}$ $(= id_S)$ is an equivalence relation.

Problem 13. Let $R$ be an equivalence relation on set $S$. Let $x, y, z \in S$. Suppose that $z \in [x] \cap [y]$. Prove that $y \in [x]$.

Problem 14. Find an integer $z \in [0, 11)$ such that $z \equiv 2^{64} \pmod{11}$.

For problems 15-17, we take $S = \{\left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right] : a, b, c, d \in \mathbb{R}\}$, the set of $2 \times 2$ matrices with real entries. We consider relation $R = \{(\left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right], \left[\begin{smallmatrix} d & a \\ b & c \end{smallmatrix}\right]) : a, b, c, d \in \mathbb{R}\}$, on $S$.

Problem 15. Prove that $R$ is a function.

Problem 16. Assume that $R$ is a function. Prove that $R$ is a bijection.

Problem 17. Compute $R \circ R \circ R$.

Problem 18. Set $T = \{a, b, c, d, e, f, g\}$. Find the width of the poset $\subseteq$ on $2^T$. Simplify your answer.

Problem 19. Let $R$ be a partial order on set $S$. Let $a, b \in S$ with $aRb$. Prove that the interval poset $[a, b]$ has a greatest element.

Problem 20. Let $S_1, S_2, S_3$ be sets, and $F_1 : S_1 \to S_2, F_2 : S_2 \to S_3$ be functions. Suppose that $F_2 \circ F_1$ is injective. Prove that $F_1$ is injective.